each is the same. IMSC only reduces the number of computations that must be carried out during the design process.

Point 7 states that IMSC uses less energy. We must distinguish between vibrational energy in the structure and energy used by the actuators in producing work. As stated earlier, the relationship between changes in vibration energy and work is independent of the control method used. However, since different actuators can have different efficiencies and different mechanical advantages, to minimize the power (or over time, energy) used by the actuators, we should use efficient (or low-cost) actuators which have good mechanical advantage more than inefficient actuators with poor mechanical advantage. Since IMSC never concerns itself with the energy going into the actuators, only the work going into the controlled modes, it is highly unlikely that IMSC would produce a minimum energy design in the sense of squared actuator effort or energy used by the actuators.

7) A numerical example is given which illustrates the advantages of IMSC. In all cases, the control system performs better when more actuators are available.

It is important to note that when quadratic performance indices are used, additional actuators will always reduce costs in a control problem in duality with the way that additional sensors always reduce uncertainty in an estimator problem (no matter how poor they are!). It is thus not unexpected that in examples where IMSC has more actuators available to it than other methods, IMSC will always appear better.

8) When costs are compared in the numerical example, the cost function used is always the one for which IMSC is optimal.

It seems meaningless to compare two control approaches if the performance index used for comparison is always exactly minimized by one of the approaches. This is like comparing apples and oranges on the criteria of which one is more "orange-like." Some "real-world" performance criteria must be used. IMSC would certainly fare very poorly if a term which was a function of the number of actuators (increased number of actuators = increased complexity and cost) was included in the cost function.

Additional practical problems exist with the IMSC method. These could be ignored if one were dealing with a purely theoretical problem, but IMSC is sold as a solution to a real problem.

1) The number of actuators must equal the number of modes.

With this constraint, the possibility of actuator failures becomes a critical problem. Must each actuator be backed up directly or must spare actuators at other locations be brought on line? Furthermore, what happens if the number of controlled modes must be increased during flight? Must spare actuators be available for this sort of contingency?

2) When IMSC is used, the B matrix (Meirovitch's notation) must be nonsingular.

What if the B matrix is nearly singular? IMSC could end up demanding nearly infinite forces from the real actuators.

3) An IMSC controller cannot be tailored to available actuators.

Mission requirements other than vibration control may produce actuator configurations incompatible with IMSC in that the actuators cannot provide the relative force distribution IMSC demands.

On a fundamental level, control systems are designed to control the outputs of the physical plant using the inputs so that certain performance specifications (which are functions of the outputs) are met. Modal equations of motion are merely a mathematical representation (an approximation, in fact) of the internal behavior of the system. The modal amplitudes and rates are thus internal (and even scaleable) variables which are only important insofar as they affect the system outputs. Keeping the internal variables decoupled through IMSC may seriously hamper efforts to control the system outputs.

References

[1]Meirovitch, L., Baruh, H., and Öz., H., "A Comparison of Control Techniques for Large Flexible Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 6, July-Aug. 1983, pp. 302-310.

[2]Vander Velde, W.E. and He, J., "Design of Space Structures Using On-Off Thrusters," *Journal of Guidance, Control, and Dynamics*, Vol. 6, Jan.-Feb. 1983, pp. 53-60.

[3]Longman, R.W. and Alfriend, K.T., "Actuator Placement from Degree of Controllability Criteria for Regular Slewing of Flexible Spacecraft," IAF Paper No. 79-95, 30th Congress of the International Astronautical Federation, Munich, Germany, Sept. 16-22, 1979.

Comment on "A Comparison of Control Techniques for Large Flexible Systems"

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IN Ref. 1, Meirovitch, Baruh, and Öz present a comparison of control system design techniques for the regulation of large flexible systems. They choose to approach the effort by grouping several quite distinct multivariable design methods under the heading "coupled controls," and then comparing them with their own technique, known as independent modal-space control (IMSC). The comparison is made on three fronts, namely 1) pole allocation, 2) linear optimal control, and 3) nonlinear on-off control. A discussion of control implementation considerations is followed by a numerical example. The authors summarize eight advantages of IMSC and yet find only one advantage among all the "coupled controls" techniques considered (namely, that the designer is not restricted, as in IMSC, to require as many actuators as there are modes in the linear system model).

Here, reexamination of the authors' three specific comparisons and further comments on control implementation will bring to light several disadvantages of IMSC which clearly temper the conclusions presented in the original work.

Pole Allocation

In discussing the various techniques for arbitrary eigenvalue assignment via state feedback, the authors point out that for multi-input systems the problem is underdetermined. That is, in general there exist many feedback gain matrices which will achieve the desired closed-loop pole placement. The remaining design freedom is used in the various techniques to achieve additional closed-loop system characteristics, for example, to obtain prescribed gain or minimum gain controllers.[2] Through conversion to a scalar input system, Kailath[3] demonstrates that all eigenvalues may be arbitrarily placed by specification of a single input, and that the remaining freedom may be used to achieve closed-loop eigenvector assignment as well.

In contrast, the IMSC method of pole placement, in addition to requiring n inputs for a $2n$-dimensional system, uses all available design freedom to achieve the desired closed-loop poles alone, and no freedom exists to specify other closed-loop system characteristics. Based on computational simplicity, the authors view IMSC as a superior approach.

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However they fail to recognize that the additional freedom inherent in pole placement in general, has been lost in their restricted formulation in which the realization of each pair of desired closed-loop poles requires a separate scalar generalized (modal) control function.

Linear Optimal Control

The solution of the linear-quadratic regulation problem is developed in Ref. 1 by first specifying the performance index

$$J=\int_0^{t_f}[w_c^T(t)Qw_c(t)+R(t)]\mathrm{d}t \tag{1}$$

where in the conventional formulation

$$R(t)=F^T(t)R_IF(t) \tag{2}$$

and $F(t)$ is the vector of *actual* control inputs. The problem admits an optimal stabilizing feedback control for arbitrary positive semidefinite Q and positive definite R_I via the solution of the standard Riccati equation.

In contrast, IMSC seeks a feedback control which minimizes Eq. (1) with

$$R(t)=f^T(t)R_2f(t) \tag{3}$$

where $f(t)$ is the vector of *modal* controls $f_r(t)$, $r=1,2,...n$. Restriction of R_2 to diagonal form and Q to two-by-two block diagonal form then decouples Eq. (1) into n second-order performance indices, one for each mode, of the form

$$J_r=\int_0^{t_f}(w_r^TQ_rw_r+\bar{R}_rf_r^2)\mathrm{d}t \tag{4}$$

The abstract generalized control input functions f_r are then obtained independently, each through the solution of a two-by-two Riccati equation. The restriction of the form of the weighting matrices therefore enables a simple elegant solution of an otherwise computationally burdensome problem, yet we will see that it creates a dilemma for the control system designer.

Contrary to the remark made in Ref. 1, the specification of R_I in the conventional formulation is straightforward and meaningful. The magnitude of the norm of R_I with respect to that of Q weights the cost of control and penalizes the system for large control effort. In addition, if we suppose R_I to be diagonal, then the elements of R_I reflect the relative penalty assigned to the control effort of each actuator.

In contrast, the designer has no guidelines for the specification of the elements $\bar{R}_r$, which each penalize the magnitude of a generalized control function and which have no direct bearing on the magnitude of control effort which will ultimately be required of the individual actuators. In fact, as first pointed out in Ref. 4, the IMSC solution may be obtained equivalently by dividing Eq. (4) by the scalar $\bar{R}_r$, yielding the performance indices

$$J_r'=\int_0^{t_f}[w_r^T(Q_r/\bar{R}_r)w_r+f_r^2]\mathrm{d}t,\qquad r=1,2,...n \tag{5}$$

Here it is evident that in IMSC all design freedom in weighting of the controls has been lost.

Nonlinear On-Off Control

The design of on-off controllers for multi-input systems is desirable for two reasons. First, the solutions of minimum time and minimum fuel control problems for linear systems with bounded controls are characterized by bang-bang or on-off control laws. Second, such control laws can be realized by relatively simple hardware (most notably, nonthrottleable gas jets for spacecraft control). The interest then lies in generating control laws which are optimal in some sense, and which are very simply implemented by on-off commands to the actuators.

IMSC cannot generate such control laws. The control laws generated by the techniques of Ref. 1 are not on-off controls, but rather quantized controls. That is, they require a variable actuator capable of producing specific finite values of control effort; moreover, the quantization levels are neither equally stepped nor are they the same for any two actuators (Ref. 1, Fig. 5).

Furthermore, optimization of either time or fuel cannot be realized by such control laws—at best, such laws will be suboptimal, requiring a conservative limitation to assure that the bounded actuators are not commanded past their physical limits.

The IMSC approach then lacks both of the desirable characteristics of on-off controls. What remains is a set of control commands in the form of step functions which one can anticipate will excite the high-order unmodeled dynamics of the system. One must ask why actuators which can implement these quantized commands would not yield better results by providing a continuously variable control input according to some other control design.

Control Implementation

In their discussion of control implementation, the authors state that "in the case of IMSC the work done to control the controlled modes does not depend on the actuator locations." A discussion of this result then leads to a much broader statement, "the actuators (sic) locations are immaterial as far as the controlled modes are concerned." While in light of Eq. (45) of their paper the first statement is accurate, the second statement is gravely misleading.

In formulating the IMSC method, the actuator location information inherent in the matrix B is removed from the control law design step. This is apparent for instance in the Riccati equation

$$\dot{K}_r=-K_rA_r-A_r^TK_r-Q_r+\bar{R}_r^{-1}K_rB_rB_r^TK_r \tag{6}$$

in which

$$B_r=[0\;\;1/\omega_r]^T \tag{7}$$

contains no actuator location information. This is a direct result of designing modal controls first, rather than determining actual controls directly. Although the actuator locations therefore do not impact the design, they do influence the *realization* of the actual controls through the equation

$$F(t)=B^{-1}f(t) \tag{8}$$

Here the actuator influence coefficients in B clearly determine the magnitude of the actual control commands in $F(t)$. Recognizing this dependence on actuator locations, Ref. 4 details several techniques for optimizing actuator locations in the context of IMSC.

Finally, it should be noted that the authors' comment that "controllability is guaranteed by definition for IMSC" is simply the result of their suggestion that "(actuator) locations should be chosen so that B is nonsingular." Controllability is therefore not *guaranteed,* but rather *required* for successful control implementation via IMSC.

Conclusions

It has been shown that the conclusions drawn from the comparison of control techniques in Ref. 1 are incomplete. To those conclusions, the following should be added: 1) both pole placement and linear optimal control suffer a significant loss of design freedom in IMSC form, 2) the practical value of a nonlinear quantized control, which is neither optimal nor simply implementable, must be questioned, 3) the influence of actuator locations on control law realization dictates that actuator placement is indeed important, and 4) the assertion of guaranteed controllability via IMSC is meaningless. Finally, and possibly more importantly, 5) the IMSC methods suffer a lack of meaningful guidelines in the choice of independent design parameters such as $\bar{R}_r$, since these parameters are related to the magnitudes of the abstract modal controls rather than actual control inputs. This was detailed here for the case of linear optimal control; it is a deficiency of the nonlinear control technique as well.

References

[1]Meirovitch, L., Baruh, H., and Öz, H., "A Comparison of Control Techniques for Large Flexible Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 6, July-Aug. 1983, pp. 302-310.

[2]Porter, B. and Crossley, T.R., *Modal Control Theory and Applications,* Taylor & Francis, Ltd., London, 1972.

[3]Kailath, T., *Linear Systems,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1970.

[4]Lindberg, R.E. and Longman, R.W., "On the Number and Placement of Actuators for Independent Modal Space Control," *Journal of Guidance, Control, and Dynamics,* Vol. 7, March-April 1984, pp. 215-221.

Comment on "A Comparison of Control Techniques for Large Flexible Systems"

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THIS paper[1] presents a control method for large space structures essentially based on dynamic decoupling strategies developed by Wonham and Morse in the late 1960's.[2] Although these ideas provide insight into the understanding of LSS behavior, they do not constitute a practical basis for control (e.g., Bryson and Hall.[3]) In addition to this lack of historical perspective on their proposed method (the contribution of the paper is not clearly defined), the authors have ignored the basic difficulties which make the LSS control problem interesting, to wit: 1) measurement noise and errors, 2) actuator location error, 3) modeling errors, 4) coupling with rigid body "modes," and the plethora of practical mechanization constraints such as actuator location and number. It has been demonstrated in numerous experiments and high-order analytical examples that control design which correctly manages such model error problems is necessary to achieve robust LSS control.[4] In this regard, the example selected as a basis for the comparison suggested in the paper's title is not sufficiently complex or representative of actual LSS control problems to justify the strength of the conclusions reached by the authors. In fact, for this problem many control design methods (including some extremely nonrobust mechanizations) can be made to work. Clearly, this is not the point of LSS controls research. The paper raises some interesting questions, but more thinking is needed on the IMSC method.

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References

[1]Meirovitch, L., Baruh, H., and Öz, H., "A Comparison of Control Techniques for Large Flexible Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 6, July-Aug. 1983, pp. 302-310.

[2]Wonham, W.M., and Morse, A.S., "Decoupling and Pole Assignment by Dynamic Compensation," *SIAM Journal on Control,* Vol. 8, Aug. 1970, pp. 317-337.

[3]Bryson, A.E., and Hall, W.E., "Modal Methods in Optimal Control Synthesis," *Advances in Dynamics and Control,* Vol. XVI, ed. by C. Leondes, Academic Press, New York, 1980, pp. 256-280.

[4]ACOSS 12 Final Report," Lockheed Missiles & Space Co. for Defense Advanced Research Projects Agency, Rome Air Development Center Technical Report, Aug. 1983.

Reply by Authors to M.A. Floyd, R.E. Lindberg, and M.G. Lyons

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THE three Technical Comments are answered individually. Because some of the arguments and references were the same, it was deemed appropriate to group the individual replies into a single one.

Reply to M.A. Floyd

1) The statement on p. 305 of Ref. 1 is not an allegation but a demonstrable fact. If the author of the Technical Comment believes that pole allocation can be used to design controls for high-order systems, say 100, then he owes it to the technical community to show the results. The same task is almost trivial if the independent modal-space control (IMSC) method is used. Indeed, as soon as the closed-loop poles have been selected, the modal gains can be obtained by means of Eq. (28), clearly a very simple computation. The fact that in IMSC the closed-loop eigenvectors are the same as the open-loop eigenvectors is not a restriction but the natural way of designing controls (see Ref. 2). Note that to change the system eigenvectors, in addition to the system eigenvalues, it takes energy, and this energy is simply wasted. Indeed, Fig. 2 of Ref. 1 shows that to control a cantilever beam by the pole allocation method, IMSC requires appreciably less energy than coupled control, although the closed-loop poles are the same in both cases. Hence, because coupled control requires more energy than IMSC for the same job, the extra energy must be wasted.

2) On page 306 of Reference 1 it is stated that if $Q = \mathrm{diag}(\omega_1^2\ \omega_1^2\ \omega_2^2\ \omega_2^2 \ldots \omega_n^2\ \omega_n^2)$, then $w_C^T Q w_C$ represents the Hamiltonian, which is ample justification. To a person

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